A SIMPLIFIED APPROACH TO HOLDING PATTERNS

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ABSTRACT. This paper proposes a simple method of executing a holding pattern for instrument pilots. It's advantages over the common procedure is that the inbound and outbound headings are easy to calculate, and the time on the outbound and inbound legs is always the same — in the case of a standard four minute pattern one minute for each leg. The common procedure results in different timings for each of the legs given a prevailing wind, and different heading corrections (crab angles) for each of the legs to remain on course. The disadvantage of the simplified method — if it may be called that — is that the racetrack pattern is abandoned in favor of the easily calculated headings and the common timing on the legs.

1. DEDICATION

This note is dedicated to Mats Moberg, friend and professional airline pilot, Widerøe.

2. INTRODUCTION

Traditionally, holding patterns follow a racetrack planform starting at a holding fix, meaning that upon entry the airplane executes a semicircular (or nearly so) track over the ground, followed by and outbound leg, again followed by a continuing approximate semicircular track, to finish with an inbound leg on the designated course. The standard pattern takes four minutes to execute, normally to the right, though sometimes Air Traffic Control [ATC] advises six minute or other timed patterns, or turns to the left. Other designations for patterns exist based on the availability of area navigation [RNAV], distance measuring equipment [DME], or global positioning system [GPS] kit on aircraft. Modern commercial and military aircraft have sophisticated electronics which make holding pattern calculations automatically, even flying the aircraft on autopilot to execute the patterns. However, such sophisticated electronics are generally not available, or only with partial capabilities, for the large number of aircraft and pilots flying outside scheduled service and without the need or budget for the most advanced equipment. As well, many pilots constitute a crew of one, even on some business jets, so anything that may reduce the workload is a welcome arrival. To note, the need for holding most frequently occurs with reduced visibility or ceiling conditions, or both, times when precise control of an aircraft is most necessary.

For background reading on the subject of holding patterns click on the url in this reference and to the citations therein. (Wikipedia 2016)

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3. Flying the simplified pattern

Let us begin with an example.

Example 3.1. *Assume that a pilot receives holding clearance for a four minute pattern while inbound at an airspeed of 160 knots to a fix on course of 360°. The wind at holding altitude is 40 knots from a direction of 060°. He knows that the outbound and inbound headings are both offsets from this wind direction. The pilot performs a simple trigonometric evaluation for the offset angle —* $\arccos(2\nu/s)$ *— where* ν *is the velocity of the wind, and* s *is the speed of the airplane. With this data, the angle is 60°. So, the pilot adds 60° to the wind direction, getting 120°, the outbound heading. He subtracts the same 60° from the wind direction, getting 000°, or 360°, the inbound heading. Upon arrival at the holding fix the pilot makes a standard rate (3° per second) right hand turn to 120°, flies straight for one minute, then continues the turn to 360°, flying straight for an additional minute to the holding fix. The second and further circuits are made the same way.*

Why does this work? Well, let us see, but first a remark.

Remark. In this example, in order to fly the simplified pattern correctly, the airplane should be on the computed inbound heading at the time of crossing the fix, in other words, should be heading 360°. This is as if the airplane were completing a pattern, ready for another. As specified however, the airplane is arriving to the fix for the first time on a *course* of 360°. In order for this to be the case the airplane would need to crab into the wind at a *heading* different from 360°. A standard wind triangle calculation tells us that this heading is 12.50°, providing a ground speed of 136.20 knots.[1] So, on arrival at the fix for the first time the pilot turns the airplane from its heading of 13° (rounded) to 120°, taking a few seconds less (about four) than would be the case for follow-up patterns when the airplane arrives at the holding fix on a *heading* of 360°. This time variation, and the distance covered for it, are negligible.

3.1. **Why does this work?** To understand the simplified pattern consider the four segments being flown in a different order, first flying a circle for two minutes (covering two of the segments,) followed by two straight legs of one minute each. This is a fantasy, of course, but it helps us understand the geometry of the pattern. We first consider the airplane moving in a stationary air mass, and then consider the effects of the moving air mass over ground locations.

For this illustration look now to Figure 1. This is diagram which follows the fantasy holding pattern as to segment order. The hypothetical wind is from the northeast, moving the air mass from point C to Point F in two minutes, and again from Point F to Point A in another two minutes. First the airplane executes a full 360° two minute turn from Point F, the holding fix. In a stationary air mass the airplane arrives back to Point F. However, with the effect of wind, the plane is actually at Point A over the ground. Then let the airplane fly two one minute straight segments in still air, first from Point A to Point B, then from Point B to Point C. Now, however, allowing for the wind, the airplane actually arrives over the holding fix F, completing the pattern.

From this construction it is only necessary to compute the headings on the two straight legs. This is easy to do, because the legs are of equal length — the distance flown in one

[1]These calculations are straightforward. "Back in the day" one made analog calculations on an E-6B calculator, on which one actually plotted the triangles. Nowadays these values are computed automatically on commercial airliners, and are easy for the flying buff to do in a spreadsheet program either by direct calculation or by an iterative method provided by a solver program. ATC personnel routinely ask pilots to report winds aloft, which they do readily from readouts computed with benefit of GPS sensors for ground speed and course.

minute, resulting in an isosceles triangle on a base of the track between the downwind and upwind reference points, A and C. They are equidistant from the holding fix by two minutes of the wind drift from the northeasterly direction. The solution is to find the angle of a right triangle (half of the isosceles triangle) which has a cosine of $2v/s$, where v is the velocity of the wind and s is the airspeed of the airplane. These angles, which are equal, are depicted in the diagram as θ (theta) and φ (phi), respectively. The numerator is proportional to the distance of the wind drift over two minutes, and the denominator is proportional to the distance the airplane flies in the air mass over one minute. In mathematical terms the angle is $\theta = \varphi = \arccos(2v/s)$, in words, θ and φ are angles which have a cosines of $2v/s$.

3.2. **The practicality of the matter.** Here now is the strategy for executing the alternative holding pattern. Approaching the fix compute the offset angle $\arccos(2v/s)$. Upon reaching the fix turn toward the right stopping to the right of the wind direction by the amount of the offset. Fly for one minute on this heading. Then continue the turn stopping to the left of the wind direction by the same amount of the offset. Fly this heading for one minute back to the holding fix. Nothing could be simpler. For a left hand pattern just exchange the words right and left, as above. It might seem counterintuitive to be heading toward the wind, with an offset, for both the outbound and inbound legs of the pattern. This necessity, however, is a consequence of being moved by the wind away from the holding fix for the entire duration of the pattern. For a six minute pattern just compute $\arccos[(3v)/(2s)]$ as the offset corresponding to two minute straight legs, reflecting the fact that the air mass is moving for six minutes. Half of this distance is proportional to $3v$, and the length of a straight leg is proportional to $2s$, the two sides of the right triangle used in the $\arccos(\cdot)$ calculation.

4. Conclusion

As you see, the alternative holding pattern is a *Gedanken* (Ger.) or *thought* experiment. Do not expect ATC to adopt this pattern any time soon, for to do so would require abandonment of their neat little world of racetrack patterns, with all their annoying calculations of crab angles and times for legs. However, if you happen to be on Visual Flight Rules someday and want to kill some time, for instance to wait for your wife to arrive at the airport just as you are rolling into the chocks, then this idea is perfect. It requires almost no thinking, and works like a dream.

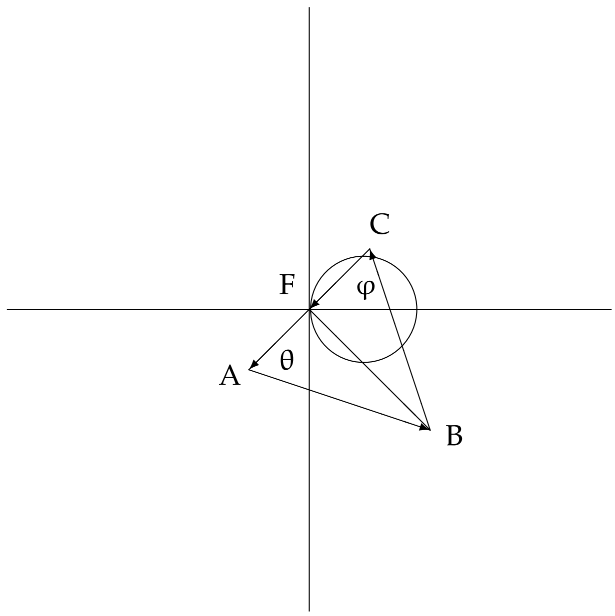

FIGURE 1. Analysis of Simplified Holding Pattern

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